

Statistical description of nuclear break-up

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Abstract. We present an overview of concepts and results obtained with statistical models in the study of nuclear multifragmentation. Conceptual differences between statistical and dynamical approaches and the selection of experimental observables for identification of these processes are outlined. New and perspective developments, like inclusion of in-medium modifications of the properties of hot primary fragments, are discussed. We list important applications of statistical multifragmentation in other fields of research.

PACS. 25.70.Pq Multifragment emission and correlations – 24.60.-k Statistical theory and fluctuations – 21.65.+f Nuclear matter

1 Introduction

Statistical models have proven to be very successful in nuclear physics. They are used for the description of nuclear decay when an equilibrated source can be identified in the reaction. The most famous example of such a source is the “compound nucleus” introduced by Niels Bohr in 1936 [1]. It was clearly seen in low-energy nuclear reactions leading to excitation energies of a few tens of MeV. It is remarkable that this concept works also for nuclear reactions induced by particles and ions of intermediate and high energies, when nuclei break up into many fragments (multifragmentation). According to the statistical hypothesis, initial dynamical interactions between nucleons lead to a re-distribution of the available energy among many degrees of freedom, and the nuclear system evolves towards equilibrium. In the most general consideration the process may be subdivided into several stages: 1) a dynamical stage leading to formation of equilibrated nuclear system, 2) the disassembly of the system into individual primary fragments, 3) the de-excitation of hot primary fragments. Below we consider these stages step by step. In this paper we give an overview of main results obtained with statistical models in multifragmentation studies, and analyze the most important problems (see also reviews [2–4]). Several hundred papers concerning multifragmentation were published during the last two decades, and we apologize that in a short review we cannot mention all works related to this field.

2 Formation of a thermalized nuclear system

At present, a number of dynamical models is used for the description of nuclear reactions at intermediate energies. The intranuclear cascade model was the first one used for realistic calculations of ensembles of highly excited residual nuclei which undergo multifragmentation, see, *e.g.* [5]. Other more sophisticated models were also used for dynamical simulations of heavy-ion reactions, such as quantum molecular dynamics (QMD), Boltzmann (Vlasov)-Uehling-Uhlenbeck (BUU, VUU) and other similar models (see, *e.g.*, ref. [6]). All dynamical models agree that the character of the dynamical evolution changes after a few rescatterings of incident nucleons, when high-energy particles (“participants”) leave the system. This can be seen from distributions of nucleon velocities and density profiles in remaining spectators [7–10]. However, the time needed for equilibration and transition to the statistical description is still under debate. This time is estimated around or less than 100 fm/c for spectator matter, however, it slightly varies in different models. Apparently, this time should be shorter for the participant zone produced in heavy-ion collisions at energies above the Fermi energy, as a result of initial compression. Parameters of the predicted equilibrated sources, *i.e.* their excitation energies, mass numbers and charges vary significantly with this time. We believe that the best strategy is to use results of the dynamical simulations as a qualitative guide line, but extract parameters of thermalized sources from the analysis of experimental data. In this case, one can avoid uncertainties of dynamical models in describing thermalization processes.

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3 Break-up of a thermalized system into hot primary fragments

3.1 Evolution from sequential decay to simultaneous break-up

After dynamical formation of a thermalized source, its further evolution depends crucially on the excitation energy and mass number. The standard compound nucleus picture is valid only at low excitation energies when sequential evaporation of light particles and fission are the dominant decay channels. Some modifications of the evaporation/fission approach were proposed in order to include emission of fragments heavier than α -particles, see *e.g.* [11–13]. However, the concept of the compound nucleus cannot be applied at high excitation energies, $E^* \gtrsim 3$ MeV/nucleon. The reason is that the time intervals between subsequent fragment emissions, estimated both within the evaporation models [14] and from experimental data [15], become very short, of the order of a few tens of fm/c. In this case there will be not enough time for the residual nucleus to reach equilibrium between subsequent emissions. Moreover, the produced fragments will be in the vicinity of each other and, therefore, should interact strongly. The rates of the particle emission calculated as for an isolated compound nucleus will not be reliable in this situation. On the other hand, the picture of a nearly simultaneous break-up in some freeze-out volume seems more justified in this case. Indeed, the time scales of less than 100 fm/c are extracted for multifragmentation reactions from experimental data [16,17]. Sophisticated dynamical calculations have also shown that a nearly simultaneous break-up into many fragments is the only possible way for the evolution of highly excited systems, *e.g.* [10,18]. Theoretical arguments in favor of a simultaneous break-up follows also from the Hartree-Fock and Thomas-Fermi calculations which predict that the compound nucleus will be unstable at high temperatures [19].

There exist several analyses of experimental data which reject the binary decay mechanism of fragment production via sequential evaporation from a compound nucleus, at high excitation energy. For example, this follows from the fact that the popular sequential GEMINI code cannot describe the multifragmentation data [20–22]. We believe that a formal reason for this failure is that the evaporation approaches always predict larger probabilities for emission of light particles (in particular, neutrons) than for intermediate mass fragments (IMFs). We mention also attempts to extend the compound nucleus picture by including its expansion within the harmonic-interaction Fermi gas (HIFGM) model [23], and within the expanding emitting-source (EES) model [24]. However, these models have the same theoretical problem with short emission times. Unfortunately, the EES model has never been compared with multifragmentation experiment in a comprehensive way since it is limited by considering emission of light IMFs with charges $Z \lesssim 10$ only.

As was shown already in early statistical model calculations, see *e.g.* [25], the entropy of the compound nucleus

dominates over entropies of multifragmentation channels at low energies, but this trend reverses at high excitation energies. This means that the evaporation/fission based models can only be used at excitation energies below the multifragmentation threshold, $E_{th} = 2\text{--}4$ MeV/nucleon. At higher excitations, a simultaneous emission must be the preferable assumption. Close to the onset of multifragmentation the most probable decay channels contain one (compound-like), or two (fission-like) fragments, and a few small fragments. With increasing excitation energy the break-up into several IMFs becomes more probable, and at very high excitation energies the decay channels with nucleons and lightest fragments (vaporization) dominate. Such evolution of nuclear-decay mechanisms is predicted by all statistical models.

3.2 Statistical models of multifragmentation

The main concepts of the statistical approach to nuclear multifragmentation have been formulated in the 1980s by Randrup *et al.* [26], Gross *et al.* (MMMC) [27], and Bondorf *et al.* (SMM) [25,28,12]. This approach is based on the assumption that the relative probabilities of different break-up channels are determined by their statistical weights, which include contributions of phase space (spatial and momentum) factors and level density of internal excitations of fragments. Different versions of the model differ in details of the description of individual fragments, Coulomb interaction and choice of statistical ensembles (grand-canonical, canonical, or microcanonical). Usually, all these details do not affect significantly qualitative features of the statistical break-up. For example, the differences in ensembles can hardly be seen in fragment distributions at high excitation energies [3], unless the observables are selected in a very special way. As was later demonstrated in experiments of many groups: ALADIN [29], EOS [30], ISIS [31], Miniball-Multics [32], INDRA [33], FASA [34], NIMROD [35] and others, equilibrated sources are indeed formed in nuclear reactions, and statistical models are very successful in describing the fragment production from them. This proves that the multifragmentation process to a large extent is controlled by the available phase space including internal excitations of fragments. Furthermore, systematic studies of such highly excited systems have brought important information about a liquid-gas phase transition in finite nuclear systems [36,37].

The success of the first statistical models has stimulated the appearance of their new versions in the next decades. The models MMM [38] and ISMM [39] are based on the same principles and use the same methods with small modifications. In the SIMON code [40], fragments evaporated from the compound nucleus are placed in a common volume in order to simulate a simultaneous break-up. There were also developments of the original models: SMM [41], and MMMC [42], bringing some improvements seen as necessary from the analysis of experimental data. An interesting mathematical development has been made in ref. [43], where a canonical version of the

SMM with simple partition weights was exactly analytically resolved by using recursive relations for the partition sum. Most models use the Boltzmann statistics, since the number of particular fragments in the freeze-out volume is typically of order 1. Calculations of ref. [44] have demonstrated that the quantum statistical effects do not play a role for all species, but only for nucleons at excitation energies and entropies characterizing multifragmentation. The same conclusion has been made in ref. [45] by direct comparisons of SMM with a quantum statistical model (QSM) [46].

As a rule, all above-mentioned statistical models give very similar results concerning the description of mean characteristics of multifragmentation. For example, the description of ALADIN experiments requires ensembles of emitting sources which in SMM, MMMC and MMM models differ within 10% of their masses and excitation energies [29,47,48]. Such an uncertainty is of the same magnitude as the precision of most experimental data. One can see some differences between the models only in more sophisticated observables. For example, the isotope properties of produced fragments, especially the isoscoping observables, may allow for better discrimination between different approaches, as well as between parameters within a specific model [49,50].

3.3 Fragment formation and freeze-out volume

In a simplified consideration, all simultaneously produced fragments are placed within a fixed freeze-out volume. It is assumed that nuclear interactions between the fragments cease at this point and that, at later times, fragments propagate independently in the mutual Coulomb field. In fact, there is a deep physical idea behind this simple picture. During the fragment formation the nucleons move in a common mean field and experience stochastic collisions. When collisions practically cease, the relatively cold group of nucleons gets trapped by the local mean field and forms fragments [51]. It is assumed that there exists a certain point in the space-time evolution which is crucial for the final fate of the system. This is a so-called “saddle” point, and the freeze-out volume provides a space for the “saddle” point configurations. According to the statistical approach, the probabilities of the fragment partitions are determined by their statistical weights at the “saddle” point. Actually, the nuclear interactions between fragments may not cease completely after the “saddle” point, however, they do not change the fragment partitions which have been decided at this point. Only when the system reaches the “scission” point the contact between the fragments is finally disrupted. This picture may be justified by the analogy with nuclear fission, where the existence of “saddle” and “scission” points is commonly accepted.

In most statistical models one assumes that “saddle” and “scission” points coincide and the statistical weight is characterized by a single freeze-out volume. On the other hand, one should distinguish the full geometrical volume and a so-called “free” volume, which is available for the

fragment translational motion in coordinate space. Due to the final size of fragments and their mutual interaction this free volume is smaller than the physical freeze-out volume, at least, by the proper volume of all produced fragments. This “excluded volume” can be included in statistical models with different prescriptions, which, however, must respect the conservation laws [52]. In the SMM there are two distinct parameters which control the free volume and the freeze-out volume. In some respects these two different volumes are introduced similar to “saddle” and “scission” points discussed above. In principal, the different volumes should be extracted from the analysis of experimental data [30,53]. Since the entropy associated with the translational motion is typically much smaller than the entropy associated with the internal excitation of fragments, uncertainties in the determination of the free volume do not affect significantly the model predictions, especially in the case of break-up into few fragments.

There are several schematic views of how the fragments are positioned in coordinate space. The most popular picture assumes expansion of uniform nuclear matter to the freeze-out volume, accompanied by its “cracking” and fragment formation. However, this picture is more appropriate for the processes with a large excitation energy and flow, and corresponds to the transition of the nucleon “gas” to the “liquid” drops by cooling during the expansion. This picture cannot be applied at energies close to the multifragmentation threshold since they are not sufficient for the essential uniform expansion of the nucleus. At $E^* \gtrsim E_{th}$ the picture of a simultaneous “fission” into several fragments seems more appropriate. One should bear in mind that for the statistical description it is not important how the system has evolved toward the “saddle” point. The only assumption in this case is that the phase space and level density factors dominate over the transition matrix elements. This explains why different models are rather consistent with each other irrespective of the way how the fragment positioning is made.

The average density which corresponds to the freeze-out volume is usually taken in the range between 1/3 and 1/10 of the normal nuclear density $\rho_0 \approx 0.15 \text{ fm}^{-3}$. In the case of thermal multifragmentation the freeze-out density can be reliably estimated from experimental data on fragment velocities since, to 80–90%, they are determined by the Coulomb acceleration after the break-up. The experimental analyses of the kinetic energies, angle and velocity correlations of the fragments indeed point to values of (0.1–0.4) ρ_0 [54,55].

3.4 Fragments in the statistical approach

Another important concept refers to “primary fragments”, *i.e.*, the fragments which are produced in the freeze-out volume. The properties of these fragments essentially determine the statistical weights of the partitions. The simplest approximation is to use the masses (or binding energies) of the nuclei from the nuclear data tables referring to cold isolated nuclei, for example, as it is done in MMMC,

or in ISMM. In order to calculate the contribution of internal fragment excitations to the statistical weight one should introduce additional assumptions concerning their level densities. For example, the MMMC prescription is 1) to limit the internal excitation of fragments by particle stable levels only (this leads to relatively cold fragments), and 2) to include in the statistical weight the contribution of secondary neutrons, which are assumed to be evaporated instantaneously from primary fragments in the freeze-out volume [2]. Randrup *et al.* [26] and MMM [38] use a Fermi-gas type approximation with a cut-off at high temperatures. In the ISMM this is done via level density expressions motivated by empirical information for isolated nuclei [39]. However, as clear from the previous discussion, the approximations used for isolated nuclei may not be true in the freeze-out volume since the fragments can still interact and, therefore, have modified properties. For example, as was noted long ago, the neutron content of primary fragments can be changed due to a reduced Coulomb interaction in the hot environment of nucleons and other fragments [12, 25, 41].

In order to include possible in-medium effects, the SMM has adopted a liquid-drop description of individual fragments ($A > 4$) extended for the case of finite temperatures and densities [3]. Smaller clusters are considered as elementary particles. At low excitation energies this description corresponds to known properties of cold nuclei, but it is generalized for the consideration of highly excited nuclei in the medium. The parameters of the liquid-drop description change as a result of interactions between the fragments leading, in particular, to modifications of bulk, surface and Coulomb terms. These parameters can be evaluated from the analysis of experimental data. As shown in ref. [50], information on possible changes in the symmetry energy of hot fragments can be extracted from the isoscaling data. This method is presently under discussion in the community. There are already experimental evidences that the symmetry energy of hot fragments in multifragmentation is significantly reduced as compared with cold nuclei [56–58]. Forthcoming experiments should clarify this conclusion.

We emphasize that in-medium modifications of fragment properties is a natural way to include the interaction between fragments within the statistical approach. Recently, an attempt has been made [59] to consider the evolution of the fragments after freeze-out within the framework of a dynamical model with explicit inclusion of nuclear interactions. As reported, this interaction results in a fusion (recombination) of primary fragments, and thus modifies the fragment partitions. However, the dynamical fragment formation after the “statistical freeze-out” leads to violation of fundamental assumptions of the statistical approach, such as the ergodicity and detailed balance principles. Generally, an application of a time-dependent approach (dynamics) to a statistical ensemble would be a controversial operation since, according to the ergodicity principle, the time average over microscopic configurations must be equivalent to the ensemble average. The dynamical consideration may be justified only for the long-range

Coulomb forces influencing fragments’ motion after their formation. Therefore, results of ref. [59] are misleading and cannot be considered as an improvement of the statistical approach.

3.5 Influence of flow on fragment formation

As was established experimentally, an “ideal” picture of thermal multifragmentation begins to fail at excitation energies of about 5–6 MeV/nucleon [60]. At higher excitations a part of the energy goes into a collective kinetic energy of the produced fragments, without thermalization. This energy is defined as the flow energy, and its share depends on the kind of reaction. For example, at thermal excitation energy of $E^* \approx 6$ MeV/nucleon, the additional flow energy is around 0.2 MeV/nucleon in hadron-induced reactions, and it is around 1.0 MeV/nucleon in central heavy-ion collisions around the Fermi energy. Since a dynamical flow itself can break matter into pieces, it is necessary to understand limits of the statistical description in the case of a strong flow.

This problem was addressed in a number of works within dynamical and lattice-gas models [10, 61–63]. Their conclusion is that a flow does not change statistical model predictions, if its energy is essentially smaller than the thermal energy. This justifies a receipt often used in statistical models, when the flow energy is included by increasing the velocities of fragments in the freeze-out volume according to the flow velocity profile [3]. This is in agreement with many experimental analyses. However, statistical models work surprisingly well even when the flow energy is comparable with the thermal energy, or even higher [42, 64]. This observation requires additional study.

3.6 Nuclear liquid-gas phase transition within statistical models

Many statistical models have demonstrated that multifragmentation is a kind of a phase transition in highly excited nuclear systems. In the SMM a link to the liquid-gas phase transition is especially strong. In particular, the surface energy of hot primary fragments is parametrized in such a way that it vanishes at a certain critical temperature. The SMM has predicted distinctive features of this phase transition in finite nuclei, such as the plateau-like anomaly in the caloric curve [3, 28], which have been later observed in experiments [36, 65]. Many other manifestations of the phase transition, such as large fluctuations and bimodality [29, 37, 66], critical behavior and even values of critical exponents [37, 67], have been investigated within this model. The experimental data are usually in agreement with the predictions.

Nevertheless, the properties of this phase transition are not yet fully understood. The critical behavior observed in experimental data can also be explained within a percolation model [68], or a Fisher’s droplet model [69], which correspond to a second-order phase transition in the vicinity of the critical point. We must note, however, that the

finiteness of the systems under investigation plays a crucial role. To connect this anomalous behavior with a real phase transition one should study it in a thermodynamical limit. Within the SMM this was done in ref. [70], where multifragmentation of an equilibrated system was identified as a first-order phase transition. The mixed phase in this case consists of an infinite liquid condensate and gas of nuclear fragments of all masses. In a finite system this mixed phase corresponds to U-shaped fragment distributions with a heaviest fragment representing the liquid phase. Thus one can connect multifragmentation of finite nuclei with the fragmentation of a very big system. This is important for the application of statistical models in astrophysical environments (neutron stars, supernova explosions), where nuclear statistical equilibrium can also be expected [71].

3.7 Relation between statistical and dynamical descriptions

One of the problems, which is highly debated now, is if dynamical models alone can describe (at least qualitatively) the same evolutionary scenario leading to equilibration and multifragmentation as assumed by statistical models. In other words, is it possible to use only a “universal” dynamical description, instead of subdividing the process into dynamical and statistical stages? Some dynamical approaches try to reach this goal starting from “first principles” like Fermionic molecular dynamics (FMD) [72], or antisymmetrized molecular dynamics (AMD) [73]. Other approaches, like QMD [6,9], NMD [10], or BNV [74] use classical equations including two-body collisions and some elements of stochasticity. In all cases dynamical simulations are more complicated and time-consuming as compared with statistical models. This is why full calculations, *e.g.* with FMD and AMD models, can only be done for relatively light systems. By using simplified receipts, like a coalescence for final fragment definition in AMD, one may reduce the computing time, but it still remains rather long. This prevents using these codes in practical calculations of nuclear fragmentation in an extended complex medium, which are required, *e.g.*, in medicine, space research, and other fields. A natural solution of this problem is to develop hybrid approaches which combine dynamical models for describing the nonequilibrium early stages of the reaction with statistical models for describing the fragmentation of equilibrated sources. In this respect, these two approaches are complementary.

One should bear in mind that the statistical and dynamical approaches are derived from different physical principles. The time-dependent dynamical approaches are based on Hamiltonian dynamics (the principle of minimal action), whereas the statistical models employ the principle of uniform population of the phase space. Actually, these two principles are not easily reducible to each other, and they represent complementary methods for describing the physical reality. There are numerous studies of the phase space population with dynamical models (see, *e.g.* [75]), which, however, have never shown the uniform

population in the limit of long times. Therefore, a decision of using statistical or dynamical approaches for the description of nuclear multifragmentation should be made after careful examination of the degree of equilibration expected in particular cases, and it can only be justified by the comparison with experiment.

There is still a large difference in details between “statistical” and “dynamical” descriptions of individual fragments as finite quantum systems. As a rule, the realistic description of clustering is difficult to achieve in dynamical models dealing with individual nucleons, but it is easily done in statistical models, considering nuclear fragments as independent degrees of freedom. In the case of equilibrated sources predictions of statistical models are usually in better agreement with experimental data. This especially concerns isospin observables. Experimental data demonstrate a characteristic trend: increasing neutron richness of intermediate mass fragments in collisions of nuclei with increasing centrality, *i.e.* with increasing excitation energy, both in the “neck region” [76] and in the equilibrated sources [77]. This trend can easily be explained in the framework of the statistical model [41]. Until now dynamical models are not very successful in describing isoscaling observables (*e.g.*, the slope coefficients) [78], while they are naturally explained within statistical approaches [49,50].

4 De-excitation and propagation of hot primary fragments

After their production in the freeze-out volume, primary fragments will propagate in the mutual Coulomb field and undergo de-excitation. It is usually assumed that the long-range Coulomb force, which has participated only partly in the fragment formation, is fully responsible for the post-freeze-out acceleration of the fragments. All statistical models solve classical Newton equations, taking into account the initial positions of fragments inside the freeze-out volume and their thermal velocities. At this stage the collective flow of fragments can also be taken into consideration.

The hot fragments will lose excitation in the course of their propagation to the detectors. There are different secondary de-excitation codes used in multifragmentation studies. The standard fission-evaporation and Fermi-break-up codes described in [12] were used in SMM [3] and MMM [48]. Another procedure, which includes GEMINI for de-excitation of big fragments, was adopted in the ISMM [39]. In the MMMC [2] a schematic model was used which takes into account only early emission of secondary neutrons. Apparently, this oversimplification is responsible for deviations of the MMMC predictions from other models in description of correlations between neutrons and charged particles [79]. It should be emphasized that most de-excitation models are based on properties of cold isolated nuclei, known from experiments at low energy. At present there is a need in more advanced models, which take into account possible in-medium modifications of primary fragments in the freeze-out volume, *e.g.*, changing

their symmetry and surface energy. An example of such a model is presented in ref. [66].

The de-excitation process depends strongly on the nuclear content of the primary fragments. For example, in the SMM at E^* slightly above E_{th} , almost all nucleons are contained in fragments, and the fraction of free nucleons is negligible. This shows an analogy with the fission process. As a result the neutron content of primary fragments is nearly the same as in the initial source. The outcome of de-excitation depends on the actual code used. Generally, in realistic statistical models most neutrons come from the secondary de-excitation stage, for example, more than 90% in the SMM. If one takes into account a reduction of the symmetry energy of primary fragments, and includes its restoration in the course of de-excitation, the neutron richness of cold final fragments will be larger than predicted by standard codes [66].

5 Conclusions

We believe that statistical models suit very well for the description of such a complicated many-body process as nuclear multifragmentation. If a thermalized source can be recognized in a nuclear reaction, the main features of multiple fragment production can be well described within the statistical approach. The success of statistical models in describing a broad range of experimental data gives us confidence that this approach will be used and further developed in the future. We especially stress two main achievements of statistical models in the theory of nuclear reactions: first, a clear understanding has been reached that sequential decay via compound nucleus must give way to a nearly simultaneous break-up of nuclei at high excitation energies; and, second, the character of this change can be interpreted as a liquid-gas-type phase transition in finite nuclear systems.

The results obtained in the nuclear multifragmentation studies can be applied in several other fields. First, the mathematical methods of the statistical multifragmentation can be used for developing thermodynamics of finite systems [80,81]. These studies were stimulated by the recent observation of extremely large fluctuations of the energy of produced fragments, which can be interpreted as the negative heat capacity [82]. At this point one can see links with cluster physics and condensed-matter physics [80]. These methods might also be useful for investigating possible phase transitions from hadronic matter to quark-gluon plasma in relativistic heavy-ion collisions.

Another conclusion is related to the fact that the multifragmentation channels take as much as 10–15% of the total cross-section in high-energy hadron-nucleus reactions, and about twice more in high-energy nucleus-nucleus collisions. Moreover, multifragmentation reactions are responsible for the production of some specific isotopes. The importance of multifragmentation reactions is now widely recognized and, in recent years, the attention given to them has risen in several domains of research. Indeed, practical calculations of fragment production and

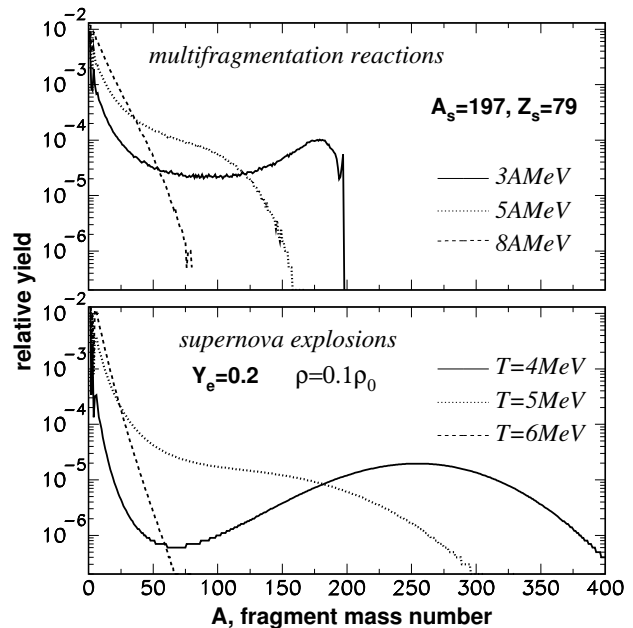


Fig. 1. Mass distributions of hot fragments calculated with SMM: top panel —for multifragmentation of Au nuclear sources at excitation energies of 3, 5, and 8 MeV per nucleon; and, bottom panel —for stellar matter at the baryon density ρ and electron fraction Y_e , typical for supernova explosions, at temperatures 4, 5, and 6 MeV.

transport in complex medium are needed for: nuclear-waste transmutation (environment protection), electro-nuclear breeding (new methods of energy production), proton and ion therapy (medical applications), radiation protection of space missions (space research). Until recently, only evaporation and fission codes have been used for describing the nuclear de-excitation. We believe that the state of the art today requires the inclusion of multifragmentation reactions in these calculations. The SMM is especially suitable for this purpose because of its multifunctional code structure: Besides the multifragmentation channels it includes also compound nucleus decays via evaporation and fission, and takes into account competition between all channels. Encouraging attempts to construct hybrid models, combining dynamical and statistical approaches, were undertaken in refs. [5, 7, 83]. The hybrid models are quite successful in describing data, including correlation observables between dynamical and statistical stages [3, 84, 85]. Several multi-purpose codes, like GEANT4 [86], have been developed to describe transport of hadrons and ions in an extended medium. The SMM was included in this code as important part responsible for fragment production.

It is important that nuclear multifragmentation reactions allow for the experimental determination of in-medium modifications of hot nuclei/fragments in hot and dense environments. This opens the unique possibility for investigating the phase diagram of nuclear matter at temperatures $T \approx 3\text{--}8\text{ MeV}$ and densities around $\rho \approx 0.1\text{--}0.3\rho_0$, which are expected in the freeze-out volume. These

studies are complementary to the previous studies of isolated nuclei existing in the matter with terrestrial densities, and at low temperatures, $T < 1\text{--}2\text{ MeV}$. The experimental information on properties of hot nuclei in dense surroundings is crucial for the construction of a reliable equation of state of stellar matter and for the modeling of the nuclear composition in supernovae [71].

As illustration of this connection we show in fig. 1 examples of fragment mass distributions produced in multifragmentation reactions (see [37,41]), and in astrophysical conditions associated with supernova type-II explosions. The calculations of nuclear compositions of stellar matter at subnuclear densities were carried out with the SMM generalized for astrophysical conditions [71,87]. One can see that the evolution of the mass distribution with excitation energy is qualitatively the same for both, the nuclear multifragmentation reactions and the supernova explosions. However, in the supernova environments much heavier and neutron-rich nuclei can be produced because of screening of their charge by surrounding electrons. This shows that studying the multifragmentation reactions in the laboratory is important for understanding how heavy elements were synthesized in the Universe.

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